Calculus 1 Workbook Solutions

Optimization

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CRITICAL POINTS

■ 1. Identify the critical point(s) of the function on the interval $[-3,2]$.

$$f(x) = x^{\frac{2}{3}}(x+2)$$

Solution:

Find $f'(x)$ and the x-values inside the given interval for which $f'(x) = 0$ or is undefined.

Rewrite the function.

$$f(x) = x^{\frac{2}{3}}(x+2)$$

$$f(x) = x^{\frac{5}{3}} + 2x^{\frac{2}{3}}$$

Find the derivative.

$$f'(x) = \frac{5}{3}x^{\frac{2}{3}} + 2 \cdot \frac{2}{3}x^{-\frac{1}{3}}$$

$$f'(x) = \frac{5}{3}\sqrt[3]{x^2} + \frac{4}{3\sqrt[3]{x}}$$

When $x = 0$, the denominator of the second fraction will be 0, which will make the derivative undefined. The derivative will also be equal to 0:

$$\frac{5}{3}\sqrt[3]{x^2} + \frac{4}{3\sqrt[3]{x}} = 0$$

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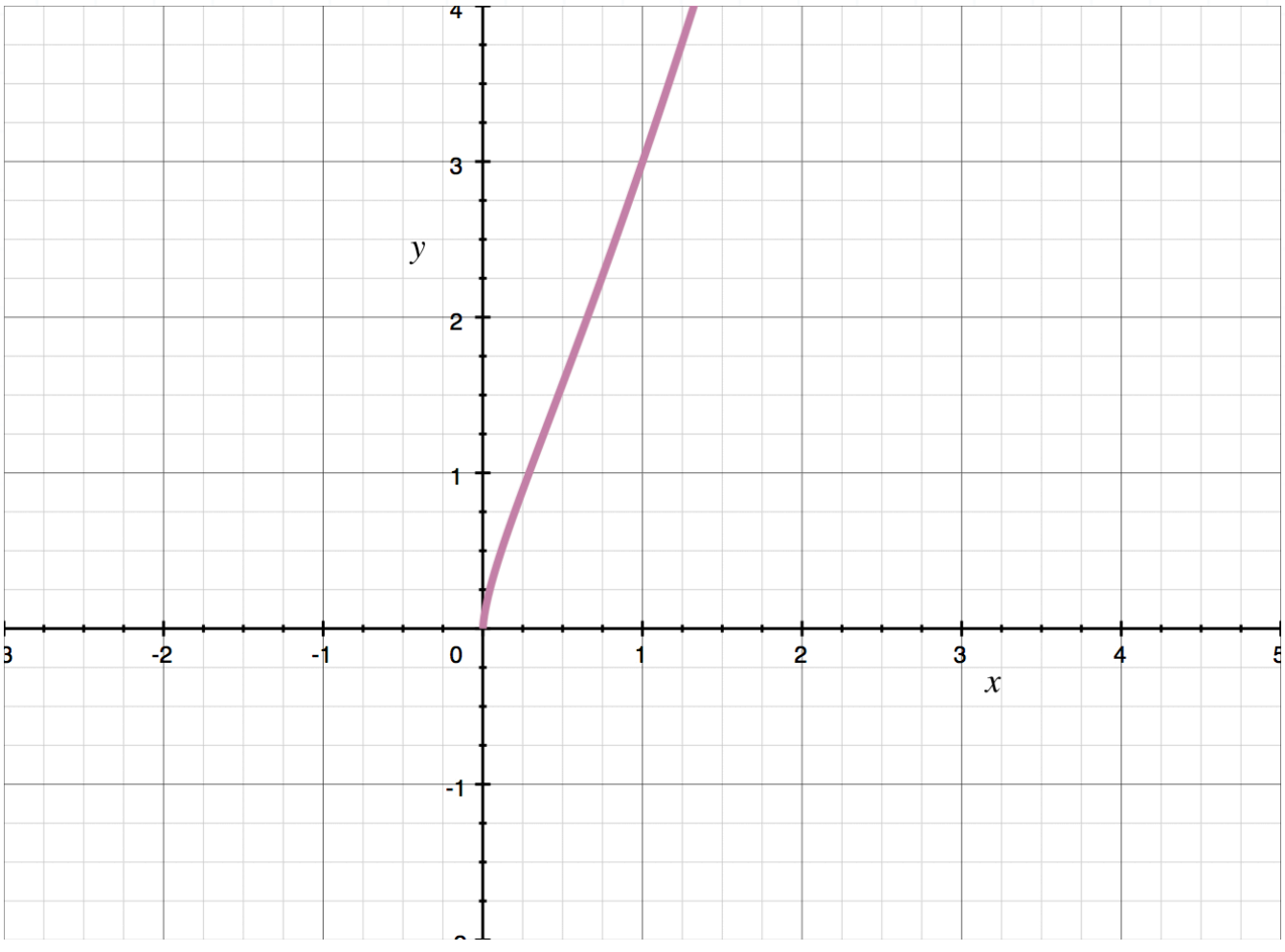
$$\frac{5}{3}\sqrt[3]{x^2} = -\frac{4}{3\sqrt[3]{x}}$$

$$5x = -4$$

$$x = -\frac{4}{5}$$

The critical numbers are therefore $x = -4/5, 0$. The graph has a horizontal tangent at $x = -4/5$ and a cusp at $x = 0$.



■ 2. Identify the critical point(s) of the function on the interval $[-2,2]$.

$$g(x) = x\sqrt{4 - x^2}$$

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Solution:

Find $g'(x)$ and the x-values inside the given interval for which $g'(x) = 0$ or is undefined.

Find the derivative.

$$g'(x) = (1)\sqrt{4 - x^2} + (x)\left(\frac{1}{2}\right)(4 - x^2)^{-\frac{1}{2}}(-2x)$$

$$g'(x) = \sqrt{4 - x^2} - x^2(4 - x^2)^{-\frac{1}{2}}$$

$$g'(x) = \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}}$$

When $x = \pm 2$, the denominator of the second fraction will be 0, which will make the derivative undefined. The derivative will also be equal to 0:

$$\sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}} = 0$$

$$\sqrt{4 - x^2} = \frac{x^2}{\sqrt{4 - x^2}}$$

$$4 - x^2 = x^2$$

$$4 = 2x^2$$

$$2 = x^2$$

$$x = \pm\sqrt{2}$$

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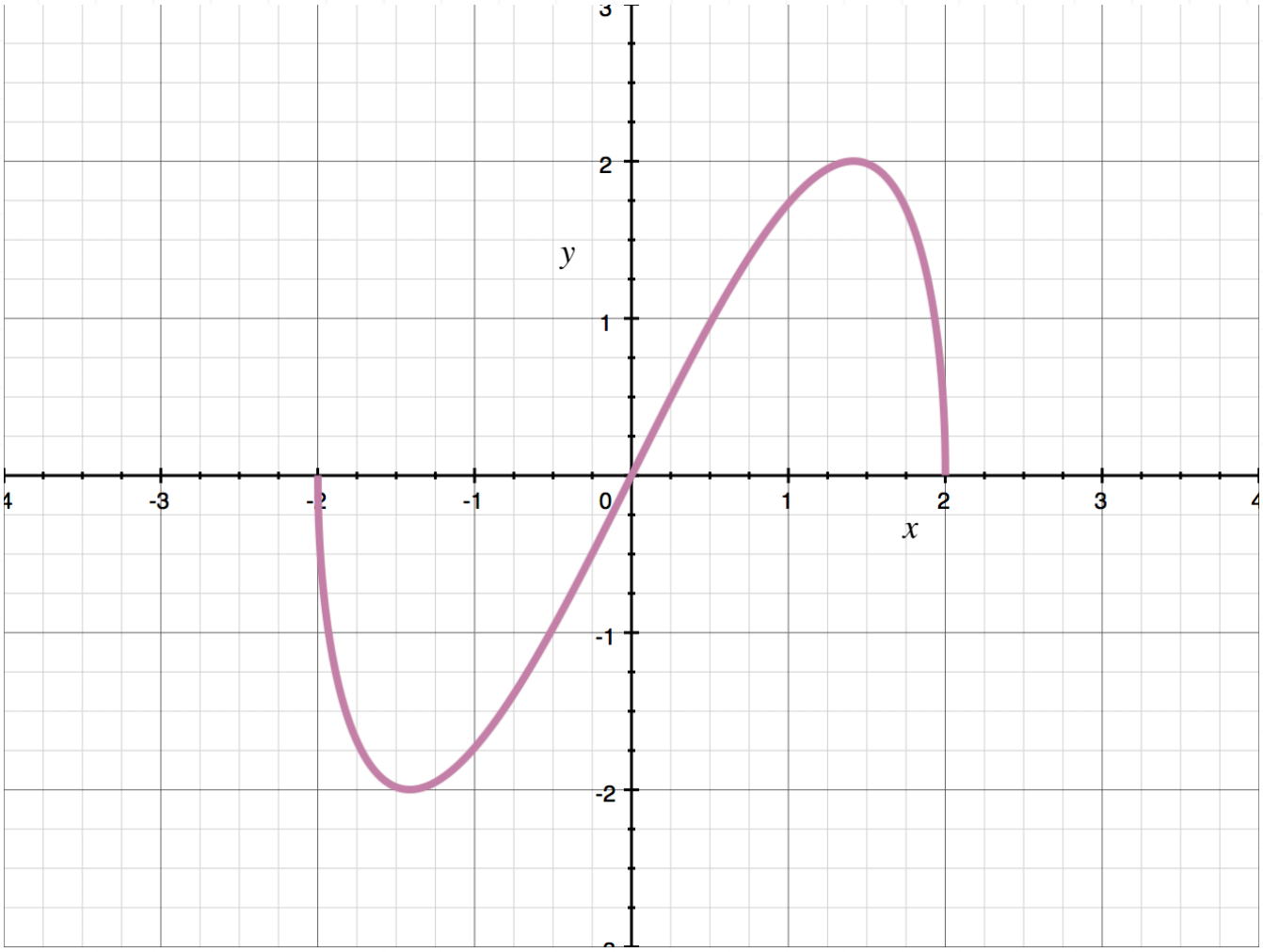
The critical numbers are therefore $x = -\pm\sqrt{2},\ \pm 2$. The graph has horizontal tangents at $x = -\pm\sqrt{2}$ and endpoint discontinuities at $x = \pm 2$.



■ 3. Identify the critical point(s) of the function on the interval $(0,4]$.

$$h(x) = \frac{1}{x} + \ln x$$

Solution:

Find $h'(x)$ and the x-values inside the given interval for which $h'(x) = 0$ or is undefined.

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Find the derivative.

$$h'(x) = -\frac{1}{x^2} + \frac{1}{x}$$

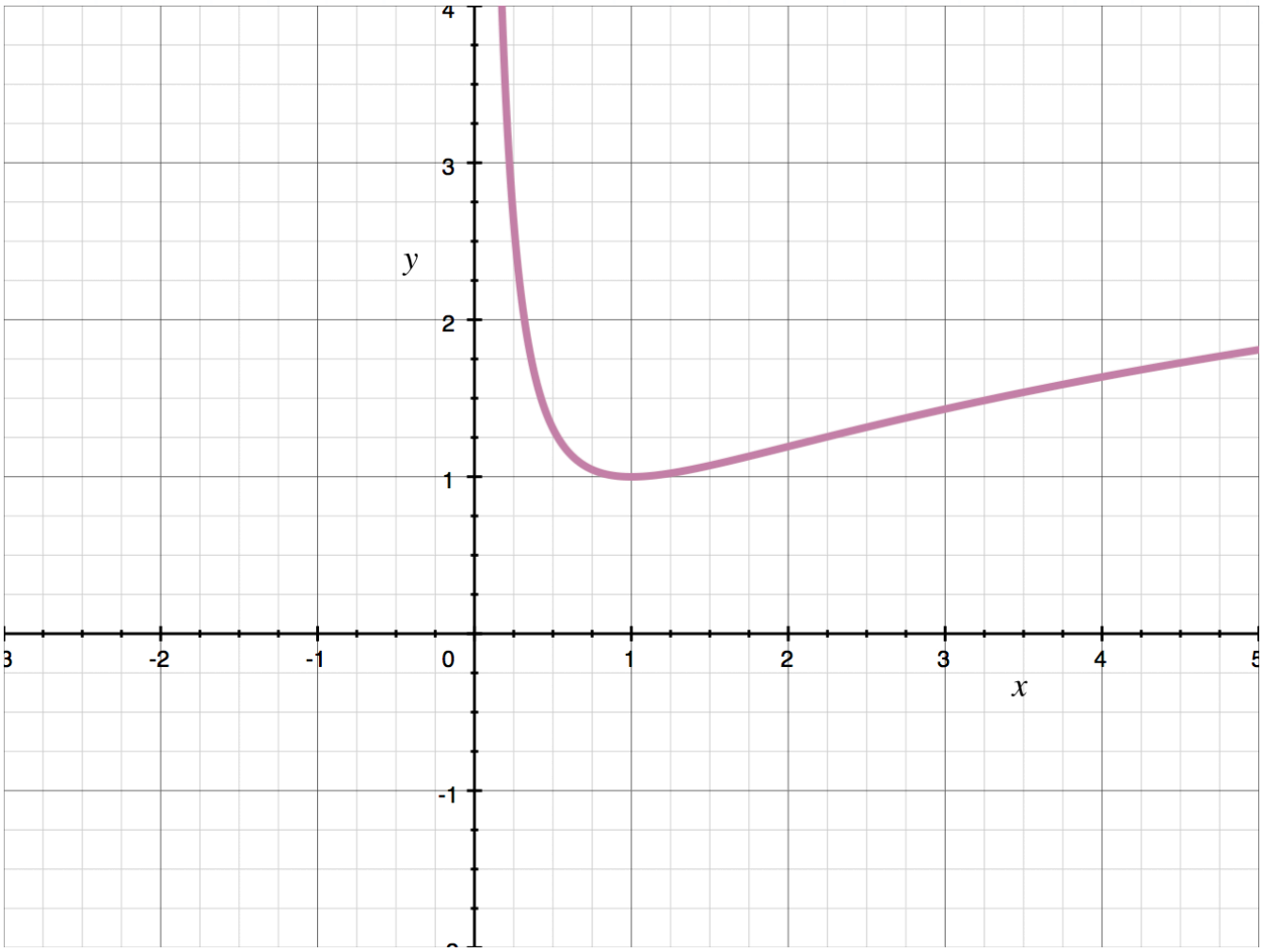
When $x = 0$, the denominator of each fraction will be 0, which will make the derivative undefined. The derivative will also be equal to 0:

$$-\frac{1}{x^2} + \frac{1}{x} = 0$$

$$\frac{1}{x} = \frac{1}{x^2}$$

$$x = 1$$

The critical numbers are therefore $x = 0, 1$. The graph has a vertical tangent at $x = 0$ and a horizontal tangent at $x = 1$.



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■ 4. Identify the critical point(s) of the function on the interval $[-3,3]$.

$$f(x) = x^3 + x^2 - 8x + 5$$

Solution:

Find $f'(x)$ and the x-values inside the given interval for which $f'(x) = 0$ or is undefined.

Find the derivative.

$$f'(x) = 3x^2 + 2x - 8$$

The derivative will be equal to 0:

$$3x^2 + 2x - 8 = 0$$

$$x^2 + \frac{2}{3}x = \frac{8}{3}$$

Complete the square in order to solve for x.

$$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{8}{3} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{24}{9} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{25}{9}$$

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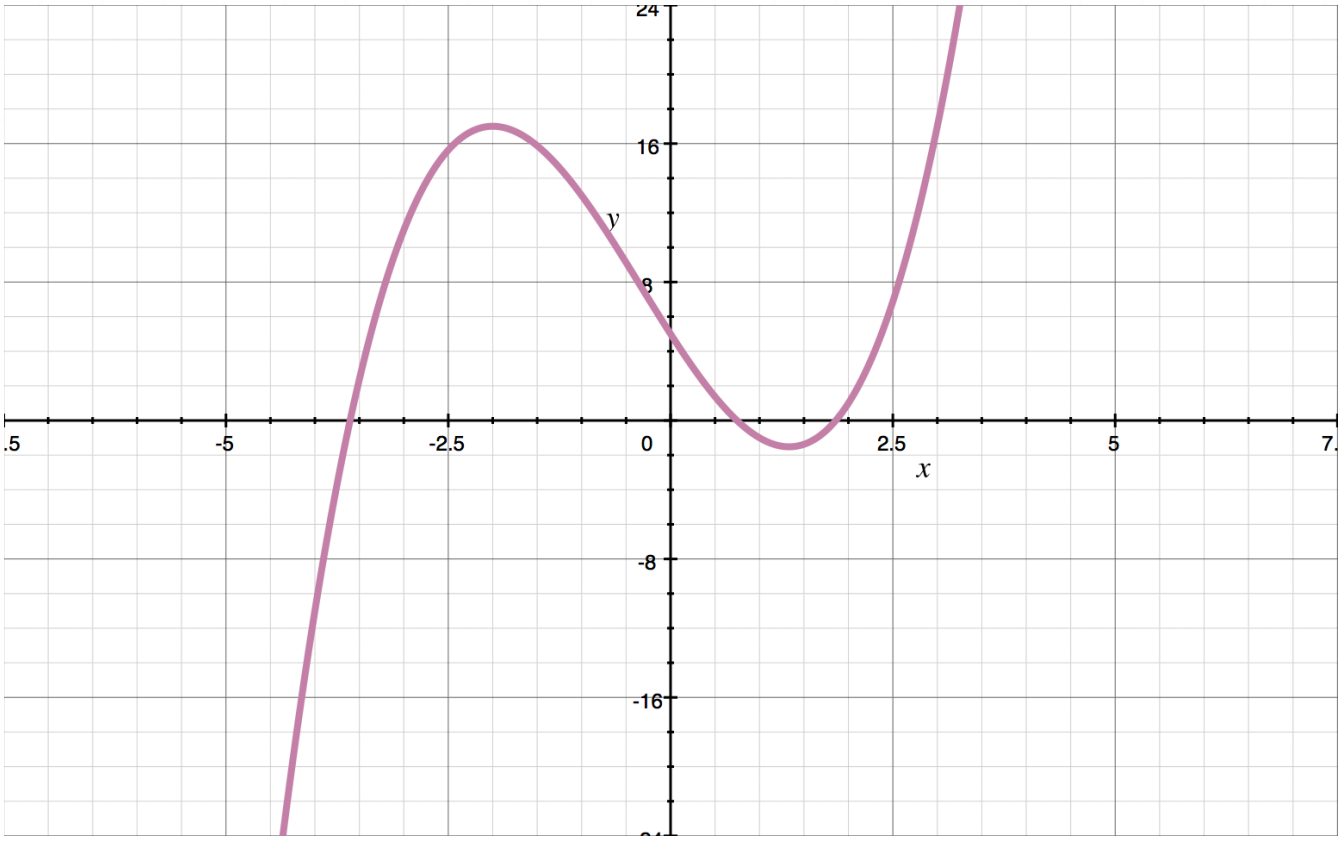
$$x + \frac{1}{3} = \pm\frac{5}{3}$$

$$x = -\frac{1 \pm 5}{3}$$

$$x = -\frac{6}{3},\ -\frac{-4}{3}$$

$$x = -2,\ \frac{4}{3}$$

The critical numbers are therefore $x = -2,\ 4/3$. The graph has horizontal tangents at $x = -2,\ 4/3$.



■ 5. Identify the critical point(s) of the function on the interval $[-1,3]$.

$$g(x) = \sqrt{3 + 2x - x^2}$$

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Solution:

Find $g'(x)$ and the x-values inside the given interval for which $g'(x) = 0$ or is undefined.

Rewrite the function.

$$g(x) = \sqrt{3 + 2x - x^2}$$

$$g(x) = (3 + 2x - x^2)^{\frac{1}{2}}$$

Find the derivative.

$$g'(x) = \frac{1}{2}(3 + 2x - x^2)^{-\frac{1}{2}}(-2x + 2)$$

$$g'(x) = (3 + 2x - x^2)^{-\frac{1}{2}}(-x + 1)$$

$$g'(x) = \frac{-x + 1}{(3 + 2x - x^2)^{\frac{1}{2}}}$$

$$g'(x) = \frac{1 - x}{\sqrt{3 + 2x - x^2}}$$

When $3 + 2x - x^2 = 0$, the denominator will be 0, which will make the derivative undefined.

$$3 + 2x - x^2 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

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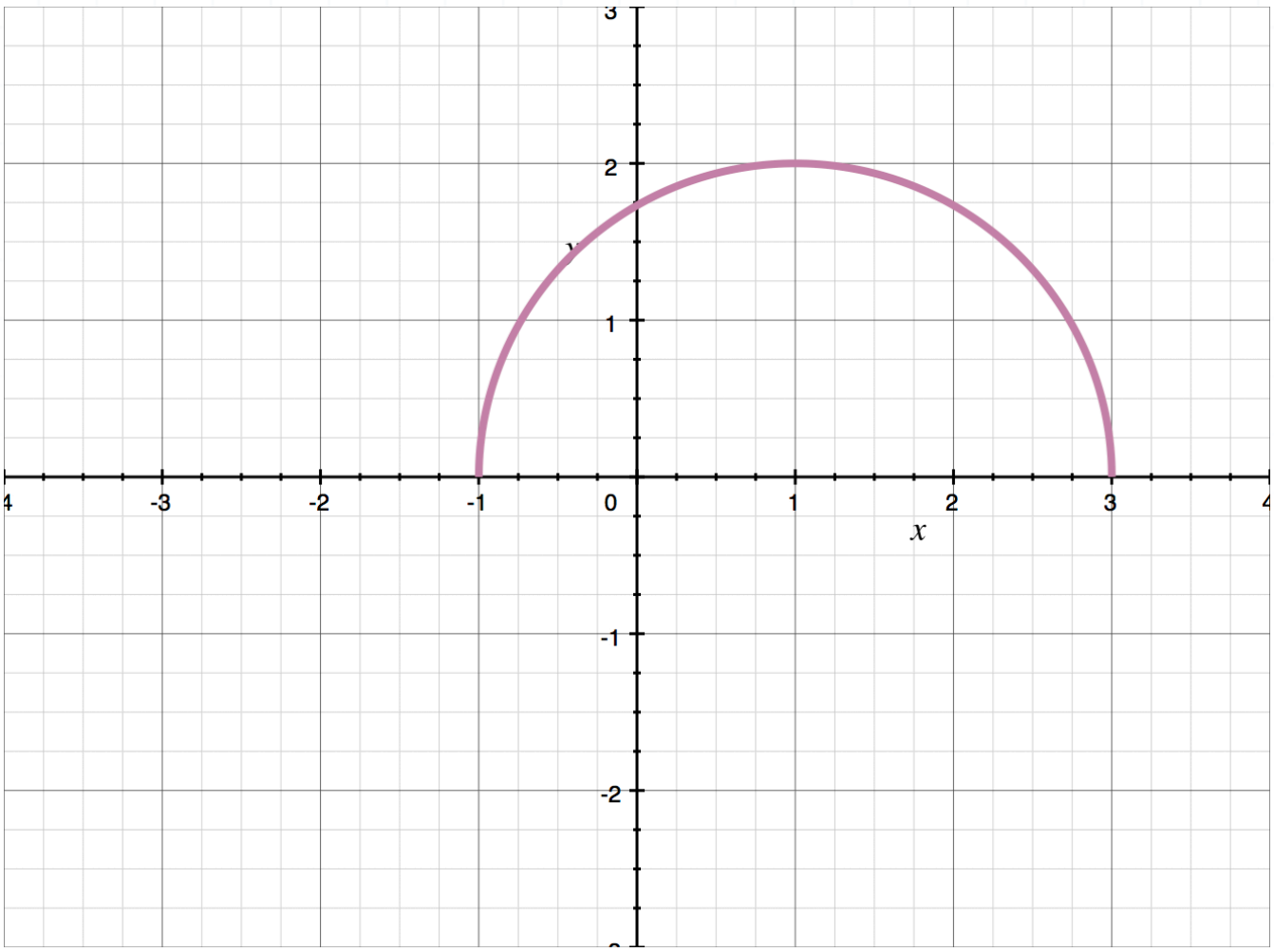
$$x = -1, 3$$

The derivative will also be equal to 0 when the numerator is equal to 0:

$$1 - x = 0$$

$$x = 1$$

The critical numbers are therefore $x = -1, 1, 3$. The graph has a horizontal tangent at $x = 1$, and endpoint discontinuities at $x = -1, 3$.



■ 6. Identify the critical point(s) of the function on the interval $[-2,2]$.

$$h(x) = \frac{3}{2}(x^4 + 4x^3 - 9x^2 + 3)$$

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Solution:

Find $h'(x)$ and the x-values inside the given interval for which $h'(x) = 0$ or is undefined.

Find the derivative.

$$h'(x) = \frac{3}{2}(4x^3 + 12x^2 - 18x)$$

The derivative will be 0 when:

$$\frac{3}{2}(4x^3 + 12x^2 - 18x) = 0$$

$$4x^3 + 12x^2 - 18x = 0$$

$$x(2x^2 + 6x - 9) = 0$$

This is true when $x = 0$ or when $2x^2 + 6x - 9 = 0$. To solve the second equation, complete the square.

$$x^2 + 3x - \frac{9}{2} = 0$$

$$x^2 + 3x = \frac{9}{2}$$

$$x^2 + 3x + \frac{9}{4} = \frac{9}{2} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{27}{4}$$

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Solve for x.

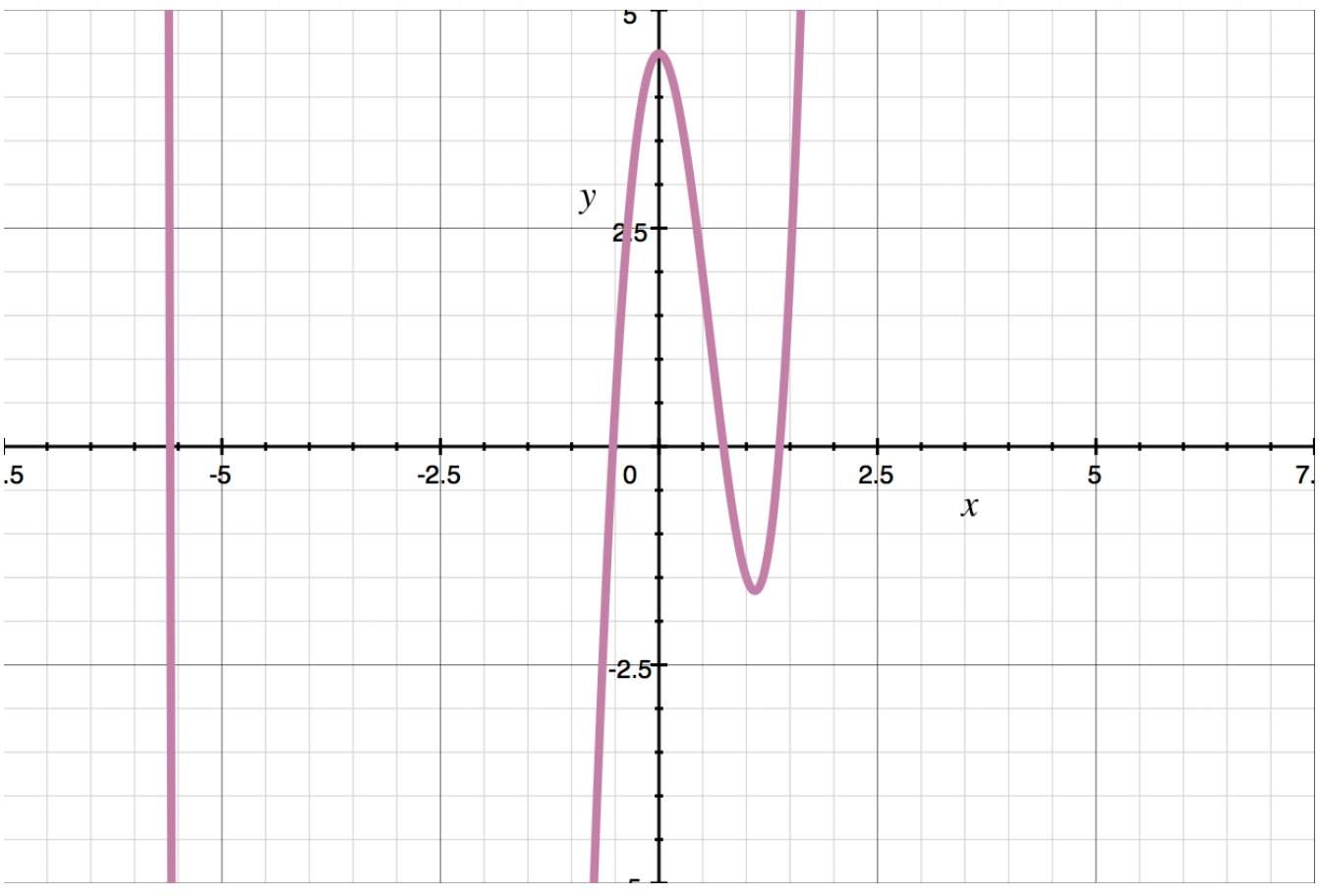
$$x + \frac{3}{2} = \pm \frac{3\sqrt{3}}{2}$$

$$x = \frac{-3 \pm 3\sqrt{3}}{2}$$

$$x = \frac{-3 + 3\sqrt{3}}{2}, \frac{-3 - 3\sqrt{3}}{2}$$

$$x = -\frac{3 - 3\sqrt{3}}{2}, -\frac{3 + 3\sqrt{3}}{2}$$

These, along with $x = 0$, are the critical numbers. All three critical points are points where the graph has horizontal tangents.



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INCREASING AND DECREASING

■ 1. Determine the intervals where the function is increasing and decreasing.

$$f(x) = \frac{5}{4}x^4 - 10x^2$$

Solution:

Find the derivative $f'(x) = 5x^3 - 20x$, then identify the critical points where $f'(x) = 0$.

$$5x^3 - 20x = 0$$

$$5x(x^2 - 4) = 0$$

$$5x(x + 2)(x - 2) = 0$$

$$x = -2,\ 0,\ 2$$

Determine where $f'(x) > 0$ or $f'(x) < 0$ by selecting a value between each critical number.

Interval	x<-2	-2<x<0	0<x<2	x>2
x	-3	-1	1	3
f'(x)	<0	>0	<0	>0
f(x)	Decreasing	Increasing	Decreasing	Increasing

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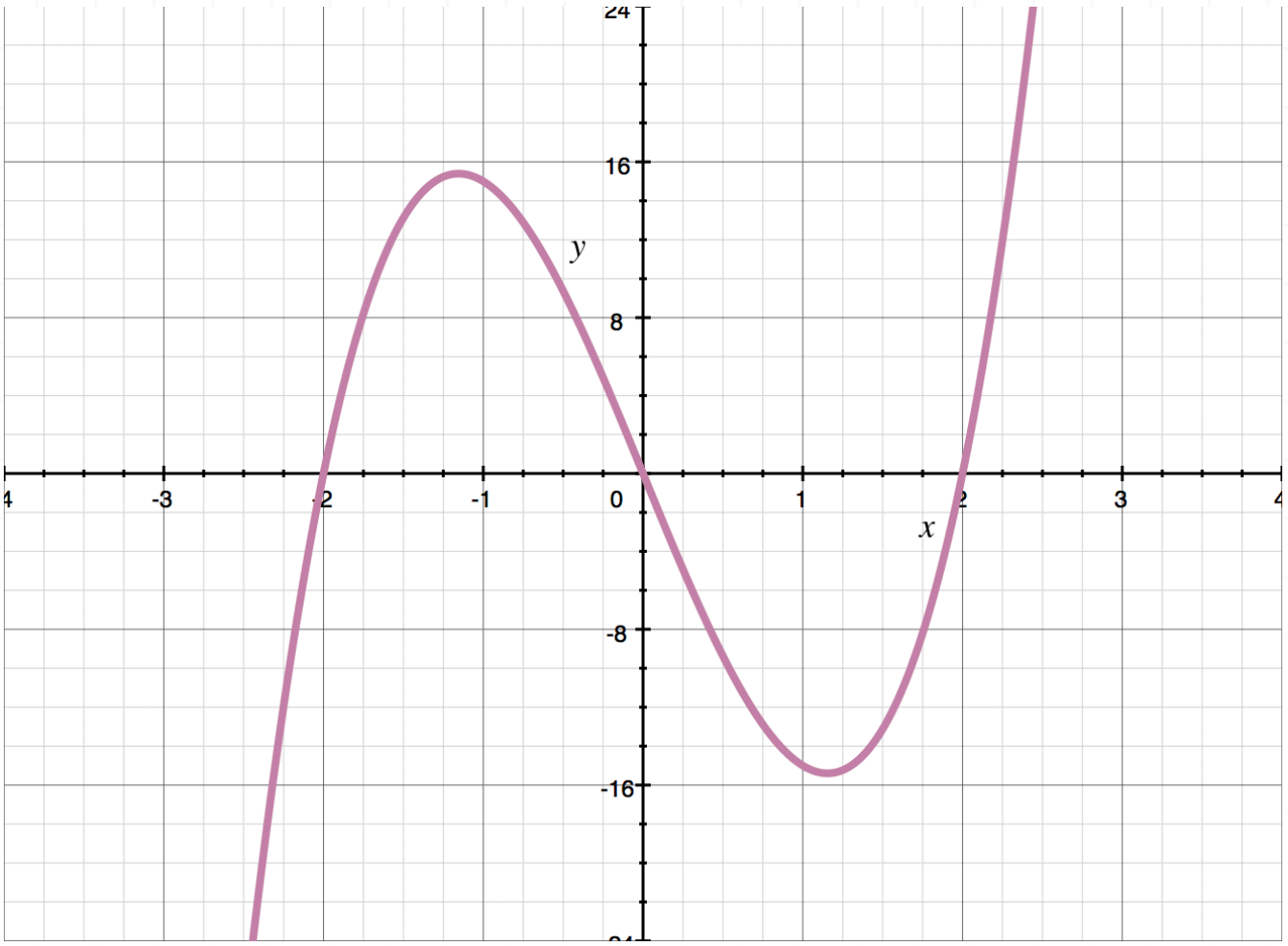
The graph of $f'(x)$ shows that $f'(x) < 0$ on $(-\infty, -2) \cup (0,2)$ and $f'(x) > 0$ on $(-2,0) \cup (2,\infty)$.



■ 2. Determine the intervals where the function is increasing and decreasing.

$g(x) = -x^3 + 2x^2 + 2$

Solution:

Find the derivative $g'(x) = -3x^2 + 4x$, then identify the critical points where $g'(x) = 0$.

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$$-3x^2 + 4x = 0$$

$$x(-3x + 4) = 0$$

$$x = 0, \frac{4}{3}$$

Determine where $g'(x) > 0$ or $g'(x) < 0$ by selecting a value between each critical number.

Interval	x<0	0<x<4/3	x>4/3
x	-1	1	2
g'(x)	<0	>0	<0
g(x)	Decreasing	Increasing	Decreasing

The graph of $g'(x)$ shows that $g'(x) < 0$ on $(-\infty,0) \cup (4/3,\infty)$ and $g'(x) > 0$ on $(0,4/3)$.

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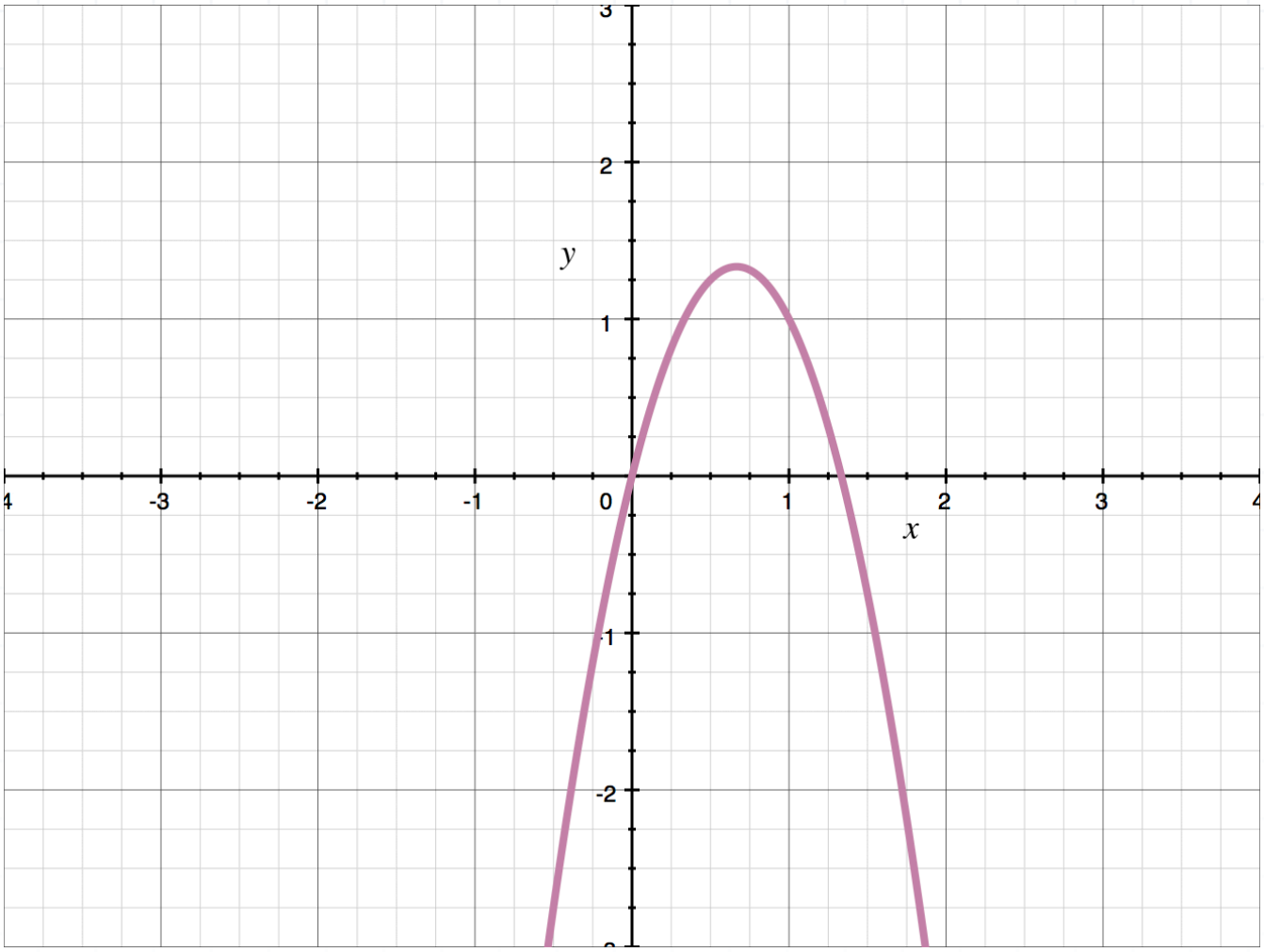

■ 3. Determine the intervals where the function $h(x) = x^3 - 11x^2 + 39x - 4$ is increasing and decreasing.

Solution:

Find the derivative $h'(x) = 3x^2 - 22x + 39$, then identify the critical points where $h'(x) = 0$.

$$3x^2 - 22x + 39 = 0$$

$$(x - 3)(3x - 13) = 0$$

$$x = 3, \frac{13}{3}$$

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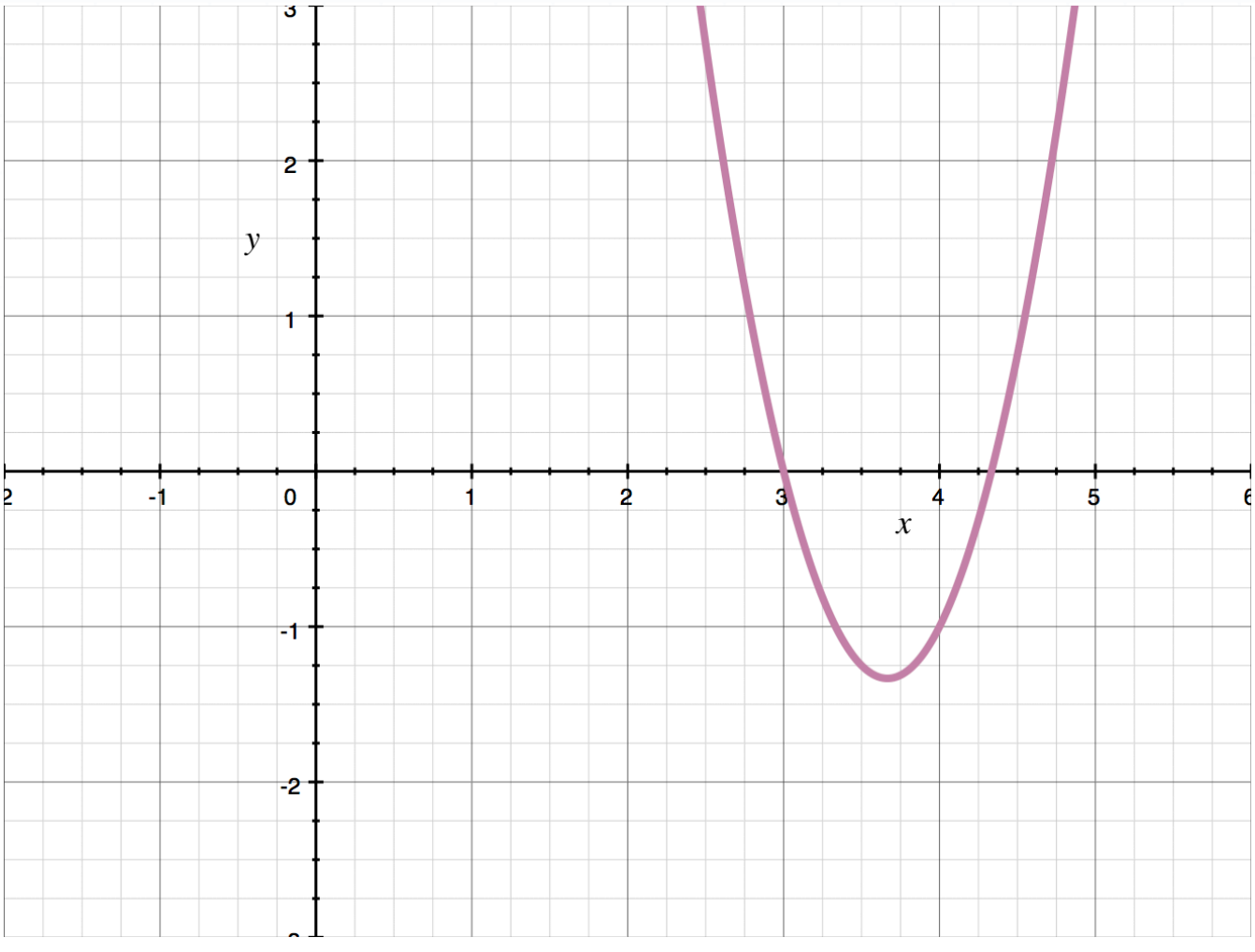
Determine where $h'(x) > 0$ or $h'(x) < 0$ by selecting a value between each critical number.

Interval	x<3	3<x<13/3	x>13/3
x	0	4	5
h'(x)	>0	<0	>0
h(x)	Increasing	Decreasing	Increasing

The graph of $h'(x)$ shows that $h'(x) > 0$ on $(-\infty,3) \cup (13/3,\infty)$ and $h'(x) < 0$ on $(3,13/3)$.



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FIRST DERIVATIVE TEST

■ 1. Use the first derivative test to find the extrema of $f(x) = 4x^3 + 21x^2 + 36x - 5$.

Solution:

Find the derivative $f'(x) = 12x^2 + 42x + 36$. Set the derivative equal to 0 and solve for x.

$$12x^2 + 42x + 36 = 0$$

$$6(2x^2 + 7x + 6) = 0$$

$$6(x + 2)(2x + 3) = 0$$

$$x = -2,\ -\frac{3}{2}$$

Determine the sign of $f'(x)$ on each side of these critical numbers by selecting a value between each number.

Interval	x<-2	x=-2	-2<x<-3/2	x=-3/2	x>-3/2
x	-4	-2	-1.8	-3/2	0
f'(x)	+	0	-	0	+
f(x)	Increasing	Maximum	Decreasing	Minimum	Increasing

The table shows that $f(x)$ has a maximum at $x = -2$, so $f(-2) = -25$ and $(-2, -25)$ is a maximum point. The table also shows that $f(x)$ has a minimum

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at $x = -3/2$, so $f(-3/2) = -101/4$ and $(-3/2, -101/4)$ is a minimum point. The graph shows the extrema:

2. Use the first derivative test to find the extrema of $g(x) = 2x^3 - 14x^2 + 22x + 3$.

Solution:

Find the derivative $g'(x) = 6x^2 - 28x + 22$. Set the derivative equal to 0 and solve for x.

$$6x^2 - 28x + 22 = 0$$

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$$2(3x^2 - 14x + 11) = 0$$

$$2(x - 1)(3x - 11) = 0$$

$$x = 1, \frac{11}{3}$$

Determine the sign of $g'(x)$ on each side of these critical numbers by selecting a value between each number.

Interval	x<1	x=1	1<x<11/3	x=11/3	x>11/3
x	-4	1	2	11/3	5
g'(x)	+	0	-	0	+
g(x)	Increasing	Maximum	Decreasing	Minimum	Increasing

The table shows that $g(x)$ has a maximum at $x = 1$, so $g(1) = 13$ and $(1,13)$ is a maximum point. The table also shows that $g(x)$ has a minimum at $x = 11/3$, so $g(11/3) = -161/27$ and $(11/3, -161/27)$ is a minimum point. The graph shows the extrema:

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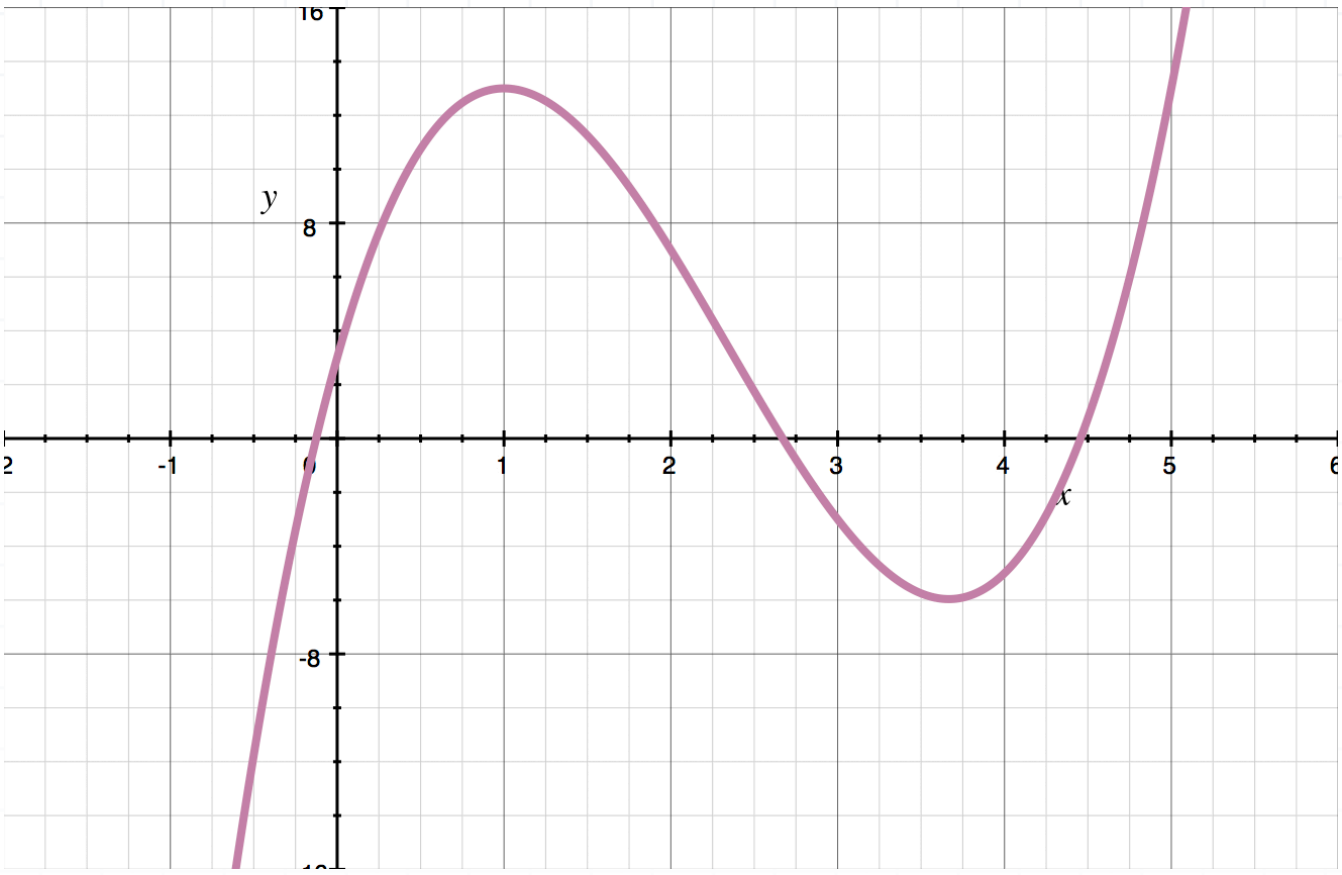

■ 3. Use the first derivative test to find the extrema of $h(x) = 2x^4 - 4x^2 + 3$.

Solution:

Find the derivative $h'(x) = 8x^3 - 8x$. Set the derivative equal to 0 and solve for x.

$$8x^3 - 8x = 0$$

$$8x(x - 1)(x + 1) = 0$$

$$x = -1,\ 0,\ 1$$

Determine the sign of $h'(x)$ on each side of these critical numbers by selecting a value between each number.

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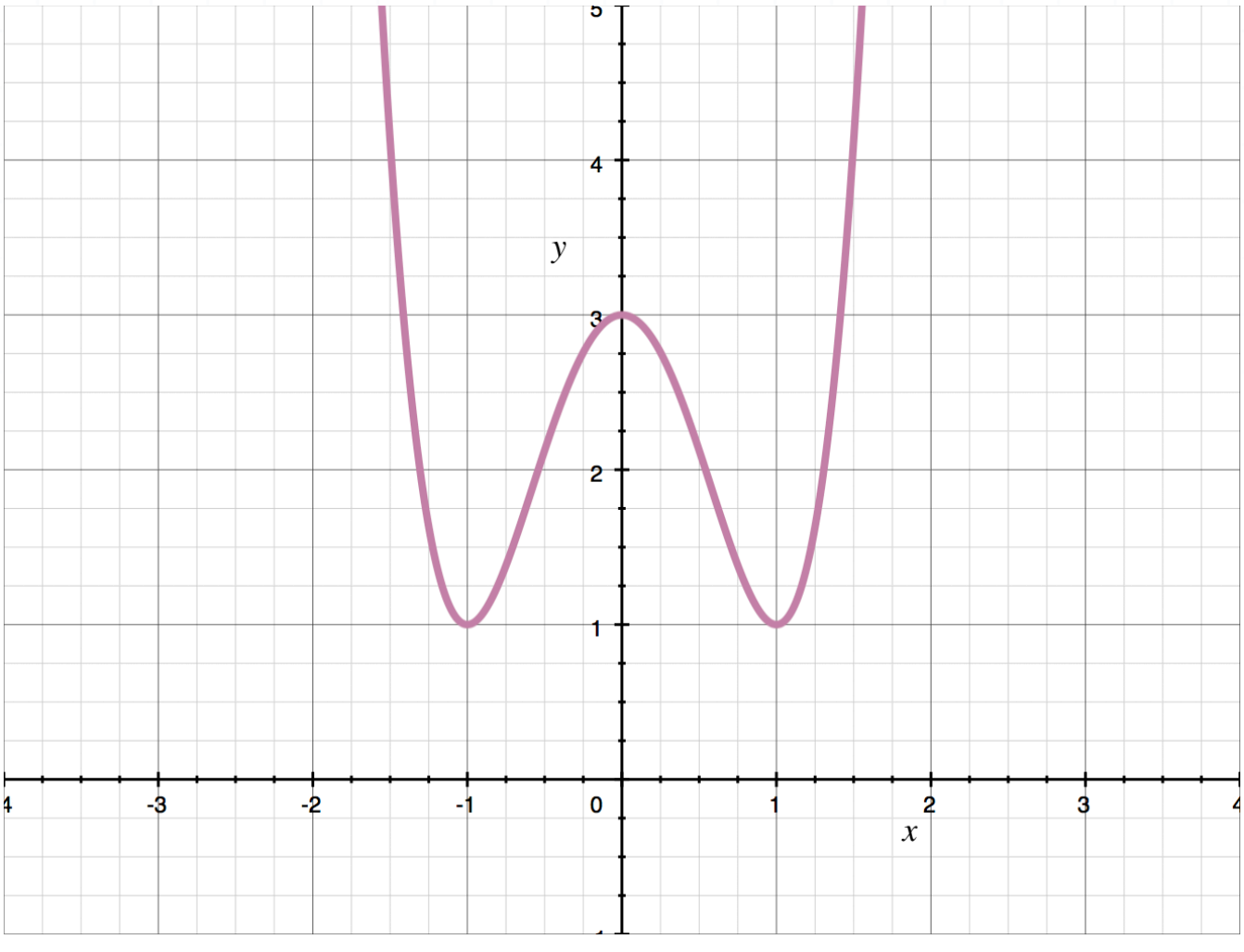
Interval	x<-1	x=-1	-1<x<0	x=0	0<x<1	x=1	x>1
x	-4	-1	-0.5	0	0.5	1	3
h'(x)	-	0	+	0	-	0	+
h(x)	Dec	Min	Inc	Max	Dec	Min	Inc

The table shows that $h(x)$ has a minimum at $x = -1$, so $h(-1) = 1$ and $(-1,1)$ is a minimum point. The table also shows that $h(x)$ has a minimum at $x = 1$, so $h(1) = 1$ and $(1,1)$ is another minimum point. The table also shows that $h(x)$ has a maximum at $x = 0$, so $h(0) = 3$ and $(0,3)$ is a maximum point. The graph shows the extrema:



■ 4. Use the first derivative test to find the extrema of $g(x) = 2x\sqrt{9 - x^2}$ on the interval $(-3,3)$.

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Solution:

Find the derivative of $g(x)$.

$$g'(x) = -\frac{2x^2}{\sqrt{9-x^2}} + 2\sqrt{9-x^2}$$

Set the derivative equal to 0 and solve for x.

$$-\frac{2x^2}{\sqrt{9-x^2}} + 2\sqrt{9-x^2} = 0$$

$$\frac{2x^2}{\sqrt{9-x^2}} = \frac{2\sqrt{9-x^2}}{1}$$

$$\frac{x^2}{\sqrt{9-x^2}} = \frac{\sqrt{9-x^2}}{1}$$

$$x^2 = 9 - x^2$$

$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \pm\sqrt{\frac{9}{2}} = \pm\frac{3}{\sqrt{2}} = \pm\frac{3\sqrt{2}}{2}$$

Determine the sign of $g'(x)$ on each side of these critical numbers by selecting a value between each number.

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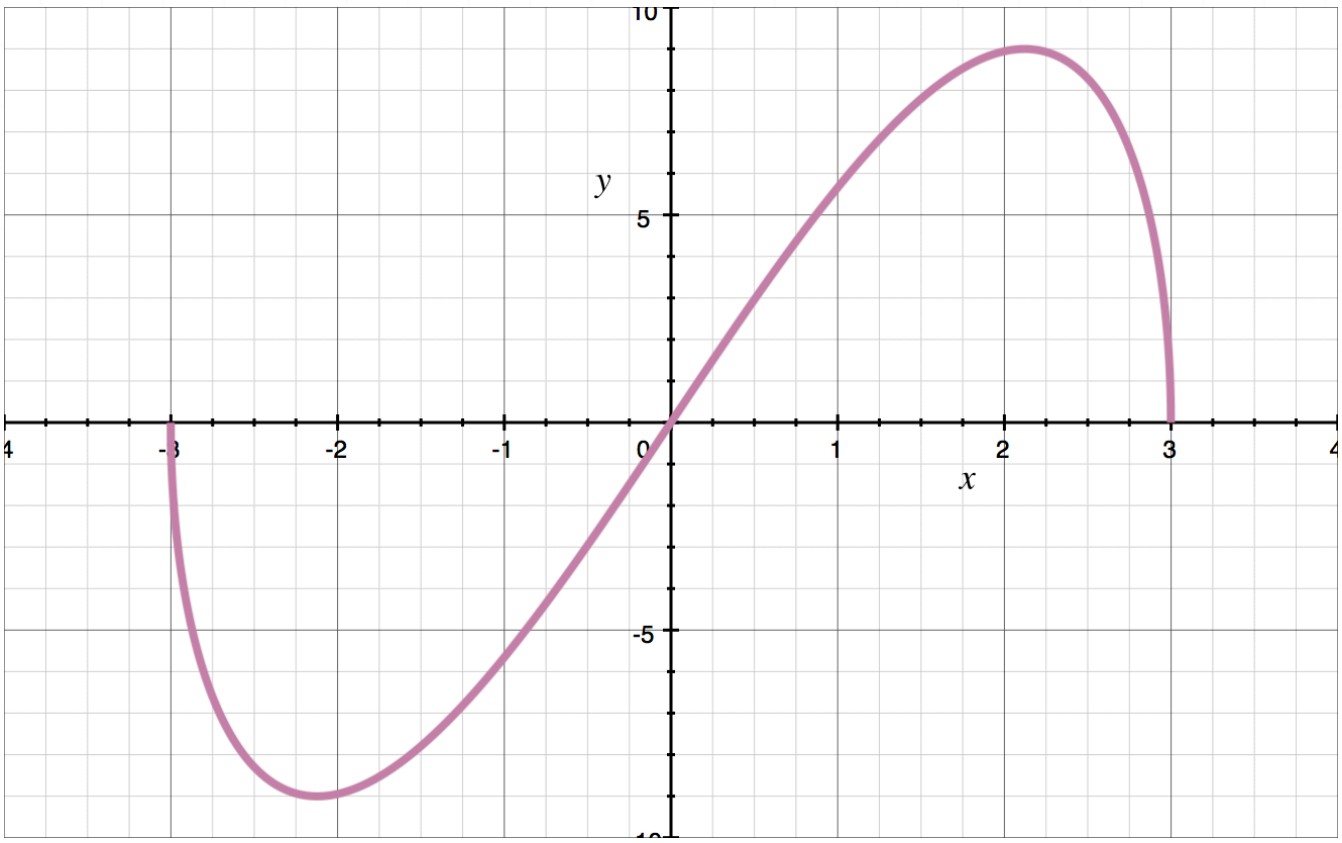
Interval	x<-2.1213	x=-2.1213	-2.1213<x<2.1213	x=2.1213	x>2.1213
x	-2.5	-2.1213	0	2.1213	2.5
g'(x)	-	0	+	0	-
g(x)	Decreasing	Minimum	Increasing	Maximum	Decreasing

The table shows that $g(x)$ has a minimum at $x = -3\sqrt{2}/2$, so $g(-3\sqrt{2}/2) = -9$ and $(-3\sqrt{2}/2, -9)$ is a minimum point. The table also shows that $g(x)$ has a maximum value at $x = 3\sqrt{2}/2$, so $g(3\sqrt{2}/2) = 9$ and $(3\sqrt{2}/2,9)$ is a maximum point. The graph shows the extrema:



5. Use the first derivative test to find the extrema of $g(x) = -2x^3 + 6x^2 - 7$.

Solution:

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Find the derivative $g'(x) = -6x^2 + 12x$. Set the derivative equal to 0 and solve for x.

$$-6x^2 + 12x = 0$$

$$-6x(x - 2) = 0$$

$$x = 0,\ 2$$

Determine the sign of $g'(x)$ on each side of these critical numbers by selecting a value between each number.

Interval	x<0	x=0	0<x<2	x=2	x>2
x	-2	0	1	2	4
g'(x)	-	0	+	0	-
g(x)	Decreasing	Minimum	Increasing	Maximum	Decreasing

The table shows that $g(x)$ has a minimum at $x = 0$, so $g(0) = -7$ and $(0, -7)$ is a minimum point. The table also shows that $g(x)$ has a maximum at $x = 2$, so $g(2) = 1$ and $(2,1)$ is a maximum point. The graph shows the extrema:

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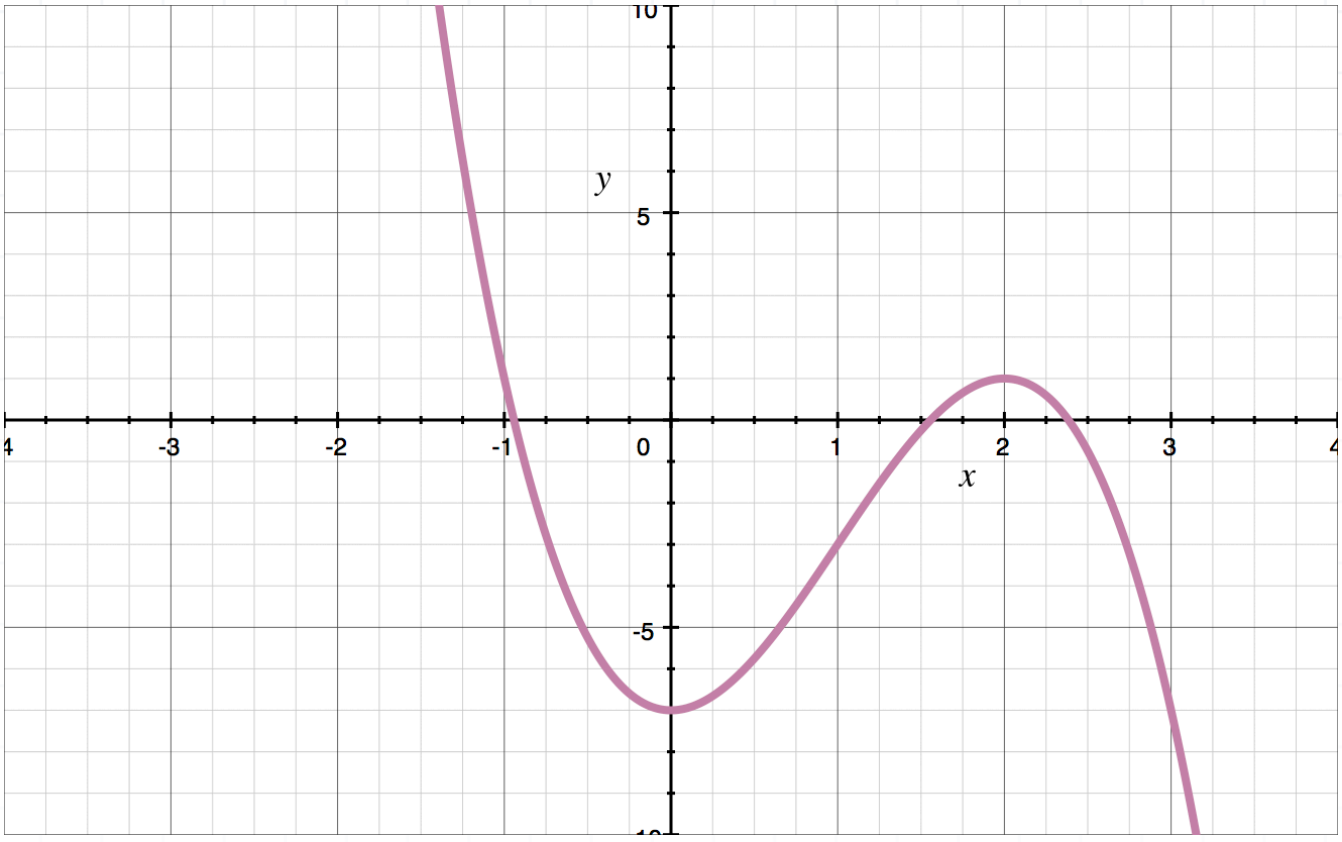

6. Use the first derivative test to find the extrema of $h(x) = 3 + 10x^2 - 5x^3$.

Solution:

Find the derivative $h'(x) = 20x - 15x^2$. Set the derivative equal to 0 and solve for x.

$$20x - 15x^2 = 0$$

$$5x(4 - 3x) = 0$$

$$x = 0, \frac{4}{3}$$

Determine the sign of $h'(x)$ on each side of these critical numbers by selecting a value between each number.

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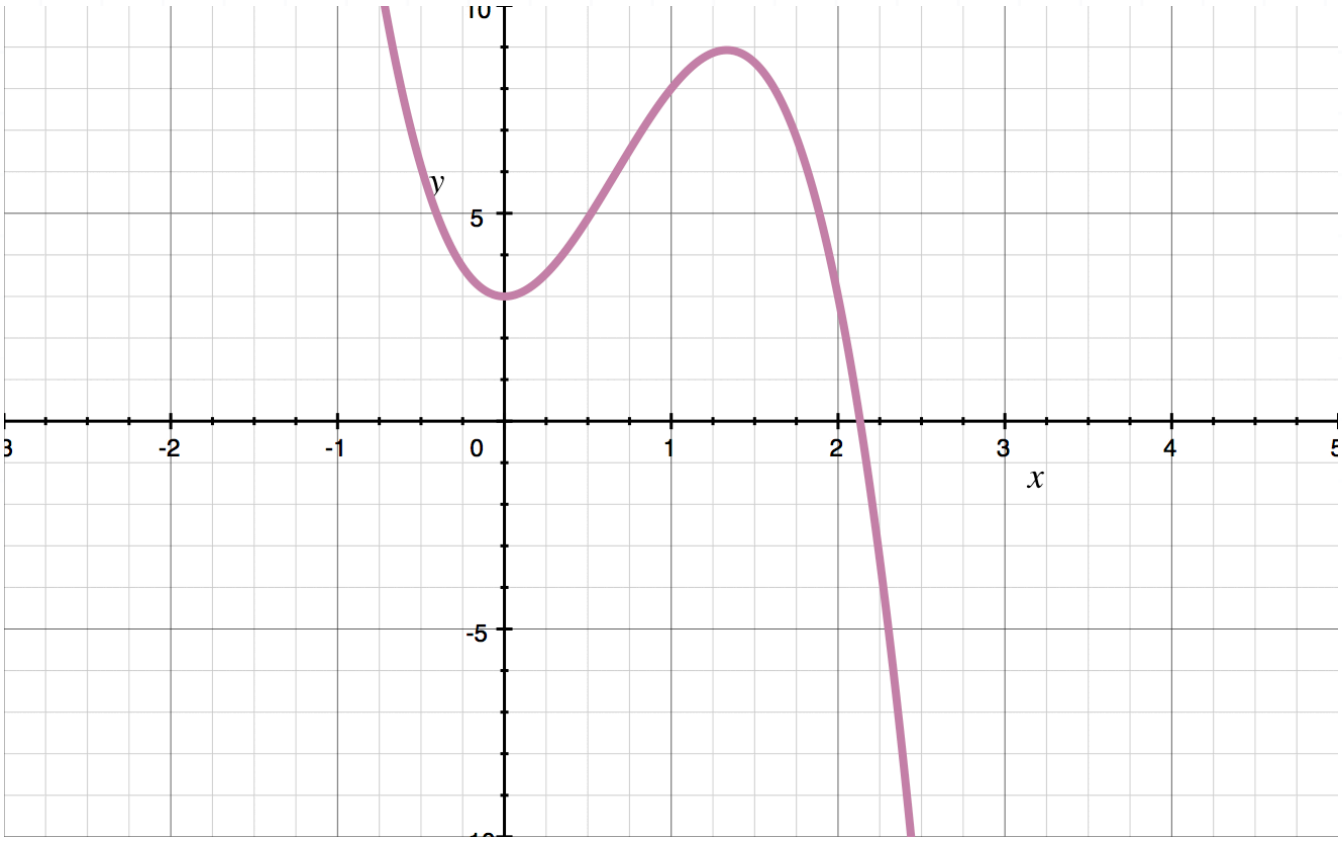
Interval	x<0	x=0	0<x<4/3	x=4/3	x>4/3
x	-2	0	1	4/3	2
h'(x)	-	0	+	0	-
h(x)	Decreasing	Minimum	Increasing	Maximum	Decreasing

The table shows that $h(x)$ has a minimum at $x = 0$, so $h(0) = 3$ and $(0,3)$ is a minimum point. The table also shows that $h(x)$ has a maximum value at $x = 4/3$, so $h(4/3) = 241/27$ and $(4/3,241/27)$ is a maximum point. The graph shows the extrema:



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CONCAVITY

■ 1. For $f(x) = x^3 - 3x^2 + 5$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

The function has an inflection point when $f''(x) = 0$.

$$6x - 6 = 0$$

$$x - 1 = 0$$

$$x = 1$$

Check values around $x = 1$.

Interval	x<1	x=1	x>1
x	-1	1	2
f”(x)	-	0	+
Concavity	Down	Inflection	Up

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The inflection point is at $x = 1$ and $f(1) = 3$, so the inflection point is $(1,3)$.
The function is concave down on $(-\infty,1)$ and concave up on $(1,\infty)$.

■ 2. For $g(x) = -x^3 + 2x^2 + 3$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$g'(x) = -3x^2 + 4x$$

$$g''(x) = -6x + 4$$

The function has an inflection point when $g''(x) = 0$.

$$-6x + 4 = 0$$

$$-6x = -4$$

$$x = \frac{2}{3}$$

Check values around $x = 2/3$.

Interval	x<2/3	x=2/3	x>2/3
x	-1	2/3	1
g”(x)	+	0	-
Concavity	Up	Inflection	Down

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The inflection point is at $x = 2/3$ and $g(2/3) = 97/27$, so the inflection point is $(2/3,97/27)$. The function is concave up on $(-\infty,2/3)$ and concave down on $(2/3,\infty)$.

■ 3. For $h(x) = x^4 + x^3 - 3x^2 + 2$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$h'(x) = 4x^3 + 3x^2 - 6x$$

$$h''(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x - 1)(x + 1)$$

The function has an inflection point when $h''(x) = 0$.

$$6(2x - 1)(x + 1) = 0$$

$$x = -1, \frac{1}{2}$$

Check values around these inflection points.

Interval	x<-1	x=-1	-1<x<1/2	x=1/2	x>1/2
x	-2	-1	0	1/2	1
h”(x)	+	0	-	0	+
Concavity	Up	Inflection	Down	Inflection	Up

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An inflection point is at $x = -1$ and $h(-1) = -1$, so an inflection point is $(-1, -1)$. Another inflection point is at $x = 1/2$ and $h(1/2) = 23/16$, so an inflection point is $(1/2,23/16)$. The function is concave up on $(-\infty, -1) \cup (1/2,\infty)$ and concave down on $(-1,1/2)$.

■ 4. For $f(x) = (x+6)^{\frac{2}{3}}$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$f'(x) = \frac{2}{3}(x+6)^{-\frac{1}{3}}$$

$$f''(x) = -\frac{2}{9}(x+6)^{-\frac{4}{3}}$$

The function has an inflection point when $f''(x) = 0$.

$$-\frac{2}{9}(x+6)^{-\frac{4}{3}} = 0$$

$$(x+6)^{-\frac{4}{3}} = 0$$

$$x = -6$$

Check values around $x = -6$.

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Interval	x<-6	x=-6	x>-6
x	-8	-6	0
f”(x)	-	DNE	-
Concavity	Down	Cusp	Down

The inflection point is at $x = -6$. While the function is undefined at that point, since there is a cusp in the graph, we can still say that the function is concave down on $(-\infty, -6) \cup (-6, \infty)$.

■ 5. For $g(x) = x^3 - 5x^2 + 7x - 3$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$g'(x) = 3x^2 - 10x + 7$$

$$g''(x) = 6x - 10$$

The function has an inflection point when $g''(x) = 0$.

$$6x - 10 = 0$$

$$6x = 10$$

$$x = \frac{5}{3}$$

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Check values around $x = 5/3$.

Interval	x<5/3	x=5/3	x>5/3
x	0	5/3	2
g''(x)	-	0	+
Concavity	Down	Inflection	Up

The inflection point is at $x = 5/3$ and $g(5/3) = -529/999$, so the inflection point is $(5/3, -529/999)$. The function is concave down on $(-\infty, 5/3)$ and concave up on $(5/3, \infty)$.

■ 6. For $f(x) = -x^3 + 6x^2 - 9x + 2$, find inflection points and identify where the function is concave up and concave down.

Solution:

Find the first and second derivatives.

$$f'(x) = -3x^2 + 12x - 9$$

$$f''(x) = -6x + 12$$

The function has an inflection point when $f''(x) = 0$.

$$-6x + 12 = 0$$

$$-6x = -12$$

$$x = 2$$

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Check values around $x = 2$.

Interval	x<2	x=2	x>2
x	-3	2	3
f''(x)	+	0	-
Concavity	Up	Inflection	Down

The inflection point is at $x = 2$ and $f(2) = 0$, so the inflection point is $(2,0)$.

The function is concave up on $(-\infty,2)$ and concave down on $(2,\infty)$.

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SECOND DERIVATIVE TEST

■ 1. Use the second derivative test to identify the extrema of $f(x) = x^3 - 12x - 2$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

The function has extrema when $f'(x) = 0$.

$$3x^2 - 12 = 0$$

$$(x + 2)(x - 2) = 0$$

$$x = -2, 2$$

Plug those values into the second derivative.

$$f''(-2) = 6(-2) = -12$$

$$f''(2) = 6(2) = 12$$

By the second derivative test, the function is concave down at $x = -2$. Since $f(-2) = 14$, $(-2,14)$ is a maximum. The function is concave up at $x = 2$. Since $f(2) = -18$, $(2, -18)$ is a minimum.

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■ 2. Use the second derivative test to identify the extrema of $g(x) = -4x^3 + 12x^2 + 5$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$g'(x) = -12x^2 + 24x$$

$$g''(x) = -24x + 24$$

The function has extrema when $g'(x) = 0$.

$$-12x^2 + 24x = 0$$

$$x(x - 2) = 0$$

$$x = 0,\ 2$$

Plug those values into the second derivative.

$$g''(0) = -24(0) + 24 = 24$$

$$g''(2) = -24(2) + 24 = -24$$

By the second derivative test, the function is concave up at $x = 0$. Since $g(0) = 5$, $(0,5)$ is a minimum. The function is concave down at $x = 2$. Since $g(2) = 21$, $(2,21)$ is a maximum.

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■ 3. Use the second derivative test to identify the extrema of $h(x) = 2x^4 - 4x^2 + 1$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$h'(x) = 8x^3 - 8x$$

$$h''(x) = 24x^2 - 8$$

The function has extrema when $h'(x) = 0$.

$$8x^3 - 8x = 0$$

$$x(x + 1)(x - 1) = 0$$

$$x = -1, 0, 1$$

Plug those values into the second derivative.

$$h''(-1) = 24(-1)^2 - 8 = 16$$

$$h''(0) = 24(0)^2 - 8 = -8$$

$$h''(1) = 24(1)^2 - 8 = 16$$

By the second derivative test, the function is concave up at $x = -1$. Since $h(-1) = -1$, $(-1, -1)$ is a minimum. The function is concave down at $x = 0$. Since $h(0) = 1$, $(0,1)$ is a maximum. The function is concave up at $x = 1$. Since $h(1) = -1$, $(1, -1)$ is a minimum.

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■ 4. Use the second derivative test to identify the extrema of $f(x) = 6x^3 - 8x^2 + 5$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$f'(x) = 18x^2 - 16x$$

$$f''(x) = 36x - 16$$

The function has extrema when $f'(x) = 0$.

$$18x^2 - 16x = 0$$

$$x(9x - 8) = 0$$

$$x = 0, \frac{8}{9}$$

Plug those values into the second derivative.

$$f''(0) = 36(0) - 16 = -16$$

$$f''\left(\frac{8}{9}\right) = 36\left(\frac{8}{9}\right) - 16 = 16$$

By the second derivative test, the function is concave down at $x = -16$. Since $f(0) = 5$, $(0,5)$ is a maximum. The function is concave up at $x = 8/9$. Since $f(8/9) = 703/243$, $(8/9,703/243)$ is a minimum.

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■ 5. Use the second derivative test to identify the extrema of $g(x) = 3x^3 - 12x^2 + 12x + 7$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$g'(x) = 9x^2 - 24x + 12$$

$$g''(x) = 18x - 24$$

The function has extrema when $g'(x) = 0$.

$$9x^2 - 24x + 12 = 0$$

$$3x^2 - 8x + 4 = 0$$

$$(3x - 2)(x - 2) = 0$$

$$x = \frac{2}{3},\ 2$$

Plug those values into the second derivative.

$$g''\left(\frac{2}{3}\right) = 18\left(\frac{2}{3}\right) - 24 = -12$$

$$g''(2) = 12$$

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By the second derivative test, the function is concave down at $x = 2/3$. Since $g(2/3) = 95/9$, $(2/3,95/9)$ is a maximum. The function is concave up at $x = 2$. Since $g(2) = 7$, $(2,7)$ is a minimum.

■ 6. Use the second derivative test to identify the extrema of $h(x) = 2x^3 - 6x + 11$ as maximum values or minimum values.

Solution:

Find the first and second derivatives.

$$h'(x) = 6x^2 - 6$$

$$h''(x) = 12x$$

The function has extrema when $h'(x) = 0$.

$$6x^2 - 6 = 0$$

$$(x + 1)(x - 1) = 0$$

$$x = -1, 1$$

Plug those values into the second derivative.

$$h''(-1) = 12(-1) = -12$$

$$h''(1) = 12(1) = 12$$

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By the second derivative test, the function is concave down at $x = -1$.

Since $h(-1) = 15$, $(-1,15)$ is a maximum. The function is concave up at $x = 1$.

Since $h(1) = 7$, $(1,7)$ is a minimum.

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